Assignment 3

# **Question 1**

1. Consider the following LP problem, in which X and Y denote the number of units of products X and Y to produce, respectively:

Maximize profit = $5X+$5Y

Subject to the constraints

2X+3Y <= 60 (resource 1)

4X+2Y <= 80 (resource 2)

X <= 18 (resource 3)

X, Y >= 0 (nonnegativity)

The Excel Sensitivity Report for this problem is shown below.

Variable Cells:

| **Cell** | **Name** | **Final Value** | **Reduced Cost** | **Objective Coefficient** | **Allowable Increase** | **Allowable Decrease** |
| --- | --- | --- | --- | --- | --- | --- |
| $B$4 | Solution value X | 15.00 | 0.00 | 5.00 | 5.00 | 1.67 |
| $C$4 | Solution value Y | 10.00 | 0.00 | 5.00 | 2.50 | 2.50 |

Constraints:

| **Cell** | **Name** | **Final Value** | **Reduced Cost** | **Objective Coefficient** | **Allowable Increase** | **Allowable Decrease** |
| --- | --- | --- | --- | --- | --- | --- |
| $D$7 | Resource 1 | 60.00 | 1.25 | 60.00 | 60.00 | 12.00 |
| $D$8 | Resource 2 | 80.00 | 0.63 | 80.00 | 8.00 | 40.00 |
| $D$9 | Resource 3 | 15.00 | 0.00 | 18.00 | 1E+30 | 3.00 |

A screenshot of a spreadsheet

AI-generated content may be incorrect.

A screenshot of a report

AI-generated content may be incorrect.

## Sub-questions

Calculate the explain what happens to the optimal solution for each of the following situations. Each question is independent of the other questions.

1. What is the optimal solution to this problem?

**The optimal solution is:**

* **Produce 15 units of product X**
* **Produce 10 units of product Y**

**This gives a total profit of:**

**Profit = 5(15) + 5(10) = $75 + $50 = $125**

1. For what ranges of values, holding all else constant, could each of the objective function coefficients be changed without changing the optimal solution?

**The profit per unit for product X can increase by up to $5 or decrease by up to $1.67 without changing the optimal solution. For product Y, its profit can increase or decrease by up to $2.50 and the solution would still remain optimal.**

1. If we could obtain one additional unit of resource 1, how would it impact profit? Over what range of RHS values could we rely upon this value?

**The shadow price of resource 1 is $1.25, so profit would increase by $1.25 per unit added.**

**This is valid within the RHS range of:**

* **Lower bound: 60 - 12 = 48**
* **Upper bound: 60 + 60 = 120**

1. If we were to give up one unit of resource 2, how would it impact profit? Over what range of RHS values could we rely upon this value?

**Giving up one unit reduces profit by $0.63 (since shadow price is 0.63).**

**Valid RHS range:**

* **Lower bound: 80 - 40 = 40**
* **Upper bound: 80 + 8 = 88**

1. If we were to increase the profit for product X by $4, how would the solution change? What would be the new solution values and the new profit?

**If the profit for product X goes from $5 to $9, it still falls within the allowable range.**

**New coefficient for X = 5 + 4 = 9**

**Since 9 is within the allowable range (3.33 to 10), the optimal solution will not change.**

**So:**

* **Solution: X = 15, Y = 10**
* **New Profit: 9(15) + 5(10) = 135 + 50 = $185**

1. If we were to decrease the profit for product Y by $2, how would the solution change? What would be the new solution values and the new profit?

**New coefficient for Y = 5 - 2 = 3**

**Since 3 is within the allowable range (2.5 to 7.5), the solution will not change.**

**So:**

* **Solution: X = 15, Y = 10**
* **New Profit: 5(15) + 3(10) = 75 + 30 = $105**

1. Suppose that two units of resource 3 were found to be unusable, how would the solution change?

**New RHS = 18 - 2 = 16**

**This is within the allowable decrease (up to 3 units), so the solution remains the same and profit stays at $125.**

1. If we were able to obtain five more units of resource 3, would you be interested in the deal? Why or why not?

**New RHS = 18 + 5 = 23**

**This is valid (allowable increase is infinite), but the shadow price is 0, meaning extra units don’t improve profit.**

**So, no, it’s not worth it.**

1. If the profit for product X was increased to $7 while at the same time the profit for product Y was reduced to $4, what would be the new solution values and the new profit?

**New coefficients: X = 7, Y = 4**

**Both values are within their respective allowable ranges (X: 3.33–10, Y: 2.5–7.5)**

**So, solution doesn’t change, only profit changes:**

**New Profit = 7(15) + 4(10) = 105 + 40 = $145**

1. Suppose you were to be offered 50 units of resource 1 at a premium of $1 each over the existing cost price for that resource. Would you purchase this? If so, by how much would your profit increase?

* **Shadow price = $1.25**
* **Profit increase = 1.25 × 50 = $62.50**
* **Cost = 50 × $1 = $50**

**Net benefit = 62.50 - 50 = $12.50, so yes, it would be profitable to buy them.**